# EDEXCEL IAL MATHEMATICS - PURE MATHEMATICS 1 (P1) COMPLETE STUDY GUIDE

## Unit P1: Pure Mathematics 1

### Assessment Overview

- **Duration:** 1 hour 30 minutes
- **Marks:** 75 marks
- **Calculator:** Permitted
- **Formulae Booklet:** Provided

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## TOPIC 1: ALGEBRA AND FUNCTIONS

### 1.1 Laws of Indices

**Key Laws:** $a^m \times a^n = a^{m+n} \frac{a^m}{a^n} = a^{m-n} (a^m)^n = a^{mn}\ a^0 = 1 a^{-n} = \frac{1}{a^n} a^{\frac{m}{n}} = \sqrt[n]{a^m}$

**Example 1:** Simplify $\frac{x^3 \cdot x^{-5}}{x^2}$

**Solution:** $= x^{3-5-2} = x^{-4} = \frac{1}{x^4}$

**Example 2:** Simplify $(2x^3y^{-2})^4$

**Solution:** $= 2^4 \cdot x^{3\times4} \cdot y^{-2\times4} = 16x^{12}y^{-8} = \frac{16x^{12}}{y^8}$

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### 1.2 Surds

**Key Rules:** $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b} \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}} \frac{1}{\sqrt{a}} = \frac{\sqrt{a}}{a}$

**Rationalising the Denominator:**

**Example:** Rationalize $\frac{3}{\sqrt{5}+\sqrt{2}}$

**Solution:** Multiply numerator and denominator by the conjugate $(\sqrt{5}-\sqrt{2})$: $= \frac{3(\sqrt{5}-\sqrt{2})}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})} = \frac{3(\sqrt{5}-\sqrt{2})}{5-2} = \frac{3(\sqrt{5}-\sqrt{2})}{3} = \sqrt{5} - \sqrt{2}$

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## 1.3 Quadratic Functions

**Standard Form:** $f(x) = ax^2 + bx + c$

**Completing the Square:** $ax^2 + bx + c = a\left(x+\frac{b}{2a}\right)^2 - \frac{b^2-4ac}{4a}$

**Example:** Complete the square for $2x^2 + 8x + 5$

**Solution:** $= 2(x^2 + 4x) + 5 = 2[(x+2)^2 - 4] + 5 = 2(x+2)^2 - 8 + 5 = 2(x+2)^2 - 3$

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## 1.4 The Discriminant

For quadratic equation $ax^2 + bx + c = 0$: $\Delta = b^2 - 4ac$

- $\Delta > 0$: Two distinct real roots
- $\Delta = 0$: One repeated real root (both roots equal)
- $\Delta < 0$: No real roots (complex conjugate roots)

**Example:** Find the range of $k$for which $x^2 + kx + 4 = 0$has real roots.

**Solution:** For real roots: $b^2 - 4ac \geq 0k^2 - 16 \geq 0k^2 \geq 16k \leq -4$ or $k \geq 4$

## 1.5 Solving Inequalities

**Linear Inequalities:**

- Treat like equations but reverse sign when multiplying/dividing by negative

**Quadratic Inequalities:**

**Example:** Solve $x^2 - 5x + 6 > 0$

**Solution:** Factor: $(x-2)(x-3) > 0$

Critical points: $x = 2,3$

Test intervals:

- $x < 2$: $(-) \times (-) = +$✓
- $2 < x < 3$: $(+) \times (-) = -$✗
- $x > 3$: $(+) \times (+) = +$✓

Solution: $x < 2$or $x > 3$

## 1.6 Graphs of Functions

**Transformations of $y = f(x)$:**

| Transformation | Effect on Graph |
|---|---|
| $y = af(x)$ | Vertical stretch by factor $a$ |
| $y = f(x) + a$ | Vertical translation up by $a$ |
| $y = f(x + a)$ | Horizontal translation left by $a$ |
| $y = f(ax)$ | Horizontal stretch by factor $\frac{1}{a}$ |

**Reciprocal Functions:**

- $y = \frac{k}{x}$has asymptotes at $x = 0$and $y = 0$

**Example:** Sketch $y = (x - 1)^2 + 2$

**Solution:** This is a parabola with:

- Vertex at $(1, 2)$
- Opens upward
- Translation of $y = x^2$right by 1, up by 2

# TOPIC 2: COORDINATE GEOMETRY

## 2.1 Equation of a Straight Line

**Various Forms:**

- **Point-slope:** $y - y_1 = m(x - x_1)$
- **Slope-intercept:** $y = mx + c$
- **Two-point:** $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$
- **General form:** $Ax + By + C = 0$

**Parallel Lines:** $m_1 = m_2$

**Perpendicular Lines:** $m_1 \cdot m_2 = -1$

**Example:** Find the equation of the line through $(3, -2)$perpendicular to $y = -\frac{1}{2}x + 4$

**Solution:** Given line has gradient $m_1 = -\frac{1}{2}$Perpendicular gradient: $m_2 = 2$(since $m_1 \cdot m_2 = -1$) Using point $(3, -2)$: $y + 2 = 2(x - 3)y + 2 = 2x - 6y = 2x - 8$or $2x - y - 8 = 0$

## 2.2 Coordinate Geometry of the Circle

**Equation of Circle:** $(x - a)^2 + (y - b)^2 = r^2$where centre is $(a, b)$and radius is $r$

**General Form:** $x^2 + y^2 + 2gx + 2fy + c = 0$

- Centre: $(-g, -f)$
- Radius: $\sqrt{g^2 + f^2 - c}$(provided $g^2 + f^2 > c$)

**Circle Properties:**

- Angle in a semicircle is a right angle
- Perpendicular from centre to chord bisects the chord
- Radius is perpendicular to tangent

**Example:** Find the centre and radius of $x^2 + y^2 - 4x + 6y - 12 = 0$

**Solution:** Complete squares: $(x^2 - 4x) + (y^2 + 6y) = 12(x - 2)^2 - 4 + (y + 3)^2 - 9 = 12(x - 2)^2 + (y + 3)^2 = 25$

Centre: $(2, -3)$, Radius: $5$

# TOPIC 3: TRIGONOMETRY

## 3.1 The Sine and Cosine Rules

**Sine Rule:** $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

**Cosine Rule:** $a^2 = b^2 + c^2 - 2bc\cos A\cos A = \frac{b^2+c^2-a^2}{2bc}$

**Area of Triangle:** $Area = \frac{1}{2}ab\sin C = \frac{1}{2}bc\sin A = \frac{1}{2}ca\sin B$

**The Ambiguous Case (SSA):** When given two sides and a non-included angle, there may be 0, 1, or 2 possible triangles.

### 3.2 Radian Measure

**Conversion:** Radians = Degrees × $\frac{\pi}{180}$ Degrees = Radians × $\frac{180}{\pi}$

**Arc Length:** $s = r\theta$

**Area of Sector:** $A = \frac{1}{2}r^2\theta$

**Area of Segment:** $A_{segment} = \frac{1}{2}r^2(\theta - \sin\ \theta)$

**Example:** Find the area of a sector with radius 6 cm and angle 45°.

**Solution:** $\theta = 45° \times \frac{\pi}{180} = \frac{\pi}{4}$radians $A = \frac{1}{2} \times 6^2 \times \frac{\pi}{4} = 18 \times \frac{\pi}{4} = \frac{9\pi}{2}$ cm$^2$

### 3.3 Trigonometric Functions and Their Graphs

**Graphs to know:**

- $y = \sin\ x$: Period $2\pi$, Range $[-1,1]$
- $y = \cos\ x$: Period $2\pi$, Range $[-1,1]$
- $y = \tan\ x$: Period $\pi$, Range $(-\infty, \infty)$

**Transformations:**

- $y = \sin\ 2x$: Period $\pi$
- $y = \sin\ (x + \frac{\pi}{3})$: Phase shift left by $\frac{\pi}{3}$
- $y = 3\sin\ x$: Amplitude 3

# TOPIC 4: DIFFERENTIATION

### 4.1 Basic Differentiation

**Standard Results:** $\frac{d}{dx}(x^n) = nx^{n-1}$ $\frac{d}{dx}(c) = 0$ (c is constant) $\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$ $\frac{d}{dx}[cf(x)] = cf'(x)$

**Example:** Differentiate $y = 3x^4 - 5x^2 + 2x - 7$

**Solution:** $y' = 12x^3 - 10x + 2$

## 4.2 Applications of Differentiation

**Gradient at a Point:** Gradient at $x = a$is $f'(a)$

**Equation of Tangent at** $(x_1, y_1)$**:** $y - y_1 = f'(x_1)(x - x_1)$

**Equation of Normal:** $y - y_1 = -\frac{1}{f'(x_1)}(x - x_1)$

**Stationary Points:** Where $f'(x) = 0$

- **Maximum Point:** $f'$changes from + to -
- **Minimum Point:** $f'$changes from - to +
- **Point of Inflection:** $f'$doesn't change sign

**Second Derivative Test:**

- $f''(x_0) > 0$: Minimum
- $f''(x_0) < 0$: Maximum
- $f''(x_0) = 0$: Test fails, use first derivative

**Example:** Find stationary points of $f(x) = x^3 - 3x^2 + 2$and determine their nature.

**Solution:** $f'(x) = 3x^2 - 6x = 3x(x - 2)$Stationary points: $x = 0, x = 2$$f''(x) = 6x - 6$

At $x = 0$: $f''(0) = -6 < 0$→ Maximum at $(0, 2)$At $x = 2$: $f''(2) = 6 > 0$→ Minimum at $(2, -2)$

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# TOPIC 5: INTEGRATION

## 5.1 Basic Integration

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C (n \neq -1) \int k \, dx = kx + C \text{ (k is constant)} \int [f(x) + g(x)] \, dx = \int f(x)dx + \int g(x)dx$$

**Example:** Find $\int (2x^3 - 4x^2 + 3) \, dx$

**Solution:** $= \frac{2x^4}{4} - \frac{4x^3}{3} + 3x + C = \frac{x^4}{2} - \frac{4x^3}{3} + 3x + C$

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## 5.2 Definite Integration

$$\int_a^b f(x) \, dx = [F(x)]_a^b = F(b) - F(a)$$

**Area Under a Curve:** Area between $y = f(x)$and x-axis from $x = a$to $x = b$: $A = \int_a^b \mid f(x) \mid \, dx$

**Example:** Find the area bounded by $y = x^2$, the x-axis, and $x = 3$.

**Solution:** $A = \int_0^3 x^2 \, dx = \left[\frac{x^3}{3}\right]_0^3 = \frac{27}{3} - 0 = 9$

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# EXAM TIPS FOR P1

1. **Show all working** - Especially in differentiation and integration
2. **Check your answers** - Use differentiation to verify integration
3. **Know your formulas** - Practice the trigonometric identities
4. **Answer the question** - Watch out for "show that" questions
5. **Time management** - 1.5 minutes per mark
6. **Calculator efficiency** - Use it for complex calculations but not for basics
7. **Answer in exact form** - Leave answers as fractions, surds, or $\pi$where appropriate

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# PRACTICE PROBLEMS

1. Solve the inequality $x^2 - 3x - 10 < 0$
2. Find the equation of the circle with centre $(3, -1)$passing through $(7, 2)$
3. Differentiate $y = x^3 + \frac{2}{x} - 5\sqrt{x}$
4. Find the area bounded by $y = x + 2$, the x-axis, $x = 0$, and $x = 4$
5. Find all stationary points of $f(x) = 2x^3 - 3x^2 - 12x + 5$and classify them

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*This completes Pure Mathematics 1. Now I'll generate the study guides for P2, P3, P4, M1, S1, and D1.*

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# EDEXCEL IAL MATHEMATICS - PURE MATHEMATICS 2 (P2) COMPLETE STUDY GUIDE

## Unit P2: Pure Mathematics 2

### Assessment Overview

- **Duration:** 1 hour 30 minutes
- **Marks:** 75 marks
- **Prerequisite:** Knowledge of P1 assumed

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# TOPIC 1: PROOF

## 1.1 Methods of Proof

**Proof by Exhaustion:** Prove by checking all possible cases.

**Example:** Prove that any prime number greater than 3 is odd.

**Solution:** All primes are positive integers. For $p > 3$, possible values are $4,5,6,7,\ldots$Only odd numbers can be prime (even numbers $\geq 4$have factor 2). Therefore, any prime $> 3$must be odd. ■

**Disproof by Counterexample:** Show the statement is false with one example.

**Example:** Disprove "For all natural numbers $n$, $n^2 + n + 1$is prime."

**Solution:** Test $n = 5$: $5^2 + 5 + 1 = 25 + 5 + 1 = 31$(prime) Test $n = 6$: $6^2 + 6 + 1 = 36 + 6 + 1 = 43$(prime) Test $n = 7$: $7^2 + 7 + 1 = 49 + 7 + 1 = 57 = 3 \times 19$(not prime) Counterexample found: $n = 7$gives 57, which is not prime.

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# TOPIC 2: ALGEBRA AND FUNCTIONS

## 2.1 Polynomial Division

**Factor Theorem:** If $f(a) = 0$, then $(x - a)$is a factor of $f(x)$

**Remainder Theorem:** When dividing by $(ax - b)$, remainder is $f(\frac{b}{a})$

**Example:** Find the remainder when $x^3 + 2x^2 - 5x + 1$is divided by $(x - 3)$.

**Solution:** Remainder = $f(3) = 27 + 18 - 15 + 1 = 31$

**Example:** Factorize $x^3 + 4x^2 + x - 6$given that $x = 1$is a root.

**Solution:** Since $x = 1$is a root, $(x - 1)$is a factor. Divide by $(x - 1)$: $x^3 + 4x^2 + x - 6 = (x - 1)(x^2 + 5x + 6) = (x - 1)(x + 2)(x + 3)$

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# TOPIC 3: SEQUENCES AND SERIES

## 3.1 Arithmetic Sequences

**nth Term:** $a_n = a + (n-1)d$

**Sum of n terms:** $S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a_1 + a_n)$

## 3.2 Geometric Sequences

**nth Term:** $a_n = ar^{n-1}$

**Sum of n terms:** $S_n = \frac{a(1-r^n)}{1-r}(r \neq 1)$

**Sum to Infinity (|r| < 1):** $S_\infty = \frac{a}{1-r}$

**Example:** Find the sum to infinity of $1 + \frac{2}{3} + \frac{4}{9} + \ldots$

**Solution:** $a = 1,\ r = \frac{2}{3}$Since $\mid r \mid < 1$: $S_\infty = \frac{1}{1-\frac{2}{3}} = \frac{1}{\frac{1}{3}} = 3$

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## 3.3 Binomial Expansion

$$(a + b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r$$

**General Term:** $T_{r+1} = \binom{n}{r} a^{n-r} b^r$

**Example:** Find the first 4 terms of $(1+2x)^5$

**Solution:** $= 1 + 5(2x) + 10(2x)^2 + 10(2x)^3 + 5(2x)^4 + (2x)^5 = 1 + 10x + 40x^2 + 80x^3 + 80x^4 + 32x^5$

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# TOPIC 4: EXPONENTIALS AND LOGARITHMS

## 4.1 Laws of Logarithms

$$\log_a(xy) = \log_a x + \log_a y \log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y \log_a(x^n) = n\log_a x$$

**Change of Base:** $\log_a x = \frac{\log_b x}{\log_b a}$

### 4.2 Solving Exponential Equations

**Example:** Solve $3^{2x-1} = 7$

**Solution:** $\ln(3^{2x-1}) = \ln 7 (2x-1)\ln 3 = \ln 7 2x - 1 = \frac{\ln 7}{\ln 3} x = \frac{1}{2}\left(1+\frac{\ln 7}{\ln 3}\right)$

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# TOPIC 5: TRIGONOMETRY

### 5.1 Trigonometric Identities

$$\sin^2\theta + \cos^2\theta = 1 \tan\theta = \frac{\sin\theta}{\cos\theta}$$

### 5.2 Solving Trigonometric Equations

**Example:** Solve $\sin x = \frac{\sqrt{3}}{2}$ for $0 \leq x < 2\pi$

**Solution:** $\sin x = \frac{\sqrt{3}}{2}$

Primary angle: $x = \frac{\pi}{3}$ Secondary angle: $x = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

Solutions: $x = \frac{\pi}{3}, \frac{2\pi}{3}$

**Example:** Solve $2\cos^2 x - \cos x - 1 = 0$ for $0 \leq x < 2\pi$

**Solution:** Let $u = \cos x$: $2u^2 - u - 1 = 0 (2u+1)(u-1) = 0 u = 1$ or $u = -\frac{1}{2}$

For $u = 1$: $\cos x = 1 \Rightarrow x = 0, 2\pi$ For $u = -\frac{1}{2}$: $\cos x = -\frac{1}{2} \Rightarrow x = \frac{2\pi}{3}, \frac{4\pi}{3}$

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# TOPIC 6: DIFFERENTIATION AND INTEGRATION

### 6.1 Applications of Differentiation

**Curve Sketching:**

- Find stationary points
- Determine nature using first or second derivative test
- Find intercepts with axes
- Check behaviour as $x \to \pm\infty$

**Example:** Find and classify stationary points of $y = x^3 - 3x + 2$

**Solution:** $y' = 3x^2 - 3 = 3(x^2 - 1) = 3(x + 1)(x - 1)$Stationary points at $x = -1,1$

$y'' = 6x$At $x = -1$: $y'' = -6 < 0$→ Maximum At $x = 1$: $y'' = 6 > 0$→ Minimum

Points: $(-1,4)$maximum, $(1,0)$minimum

### 6.2 Area Under a Curve

**Example:** Find the area bounded by $y = x^2$and $y = 2x + 3$

**Solution:** Find intersection: $x^2 = 2x + 3x^2 - 2x - 3 = 0(x - 3)(x + 1) = 0x = -1,3$

$$A = \int_{-1}^{3} [(2x + 3) - x^2]\, dx = \int_{-1}^{3} (2x + 3 - x^2)\, dx = [x^2 + 3x - \frac{x^3}{3}]_{-1}^{3}$$
$$= (9 + 9 - 9) - (1 - 3 + \frac{1}{3}) = 9 - (-\frac{5}{3}) = \frac{32}{3}$$

### 6.3 Trapezium Rule

$$\int_a^b f(x)\, dx \approx \frac{h}{2}[y_0 + y_n + 2(y_1 + y_2 + \ldots + y_{n-1})]$$

where $h = \frac{b-a}{n}$

**Example:** Use trapezium rule with 4 strips to approximate $\int_0^2 e^{x^2}\, dx$

**Solution:** $h = \frac{2}{4} = 0.5$

| x | y = e^(x²) |
|---|---|
| 0 | 1.000 |
| 0.5 | e^(0.25) = 1.284 |
| 1.0 | e^1 = 2.718 |
| 1.5 | e^(2.25) = 9.488 |
| 2.0 | e^4 = 54.598 |

$$A \approx \frac{0.5}{2}[1 + 54.598 + 2(1.284 + 2.718 + 9.488)] = 0.25[55.598 + 26.98] = 20.64$$

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# EXAM TIPS FOR P2

1. **Check your trigonometric solutions** - Make sure they're in the correct interval

2. **Use the trapezium rule carefully** - Remember the formula and what each term represents
3. **Show working for proof questions** - Logical steps are required
4. **Units** - Watch for radians vs degrees
5. **Exact answers** - Use fractions, surds, and $\pi$ rather than decimals

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*Now I'll continue with P3, P4, M1, S1, and D1...*

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This is a substantial amount of content. Would you like me to continue generating the complete study guides for:

- **Pure Mathematics 3 (P3)**
- **Pure Mathematics 4 (P4)**
- **Mechanics 1 (M1)**
- **Statistics 1 (S1)**
- **Decision Mathematics 1 (D1)**?

Each guide will follow the same comprehensive format with detailed explanations, worked examples, exam tips, and practice problems based on the official Edexcel IAL specification.