# EDEXCEL IAL MATHEMATICS - MECHANICS 1 (M1) COMPLETE STUDY GUIDE

## Unit M1: Mechanics 1

### Assessment Overview

- **Duration:** 1 hour 30 minutes
- **Marks:** 75 marks
- **Prerequisite:** Knowledge of P1 and P2, vectors in two dimensions
- **Calculator:** Permitted
- **Formulae Booklet:** Provided

### Key Formulae (Provided in Exam)

**Momentum:** $p = mv$

**Impulse:** $I = mv - mu$

**Constant Acceleration Equations:**

$$v = u + ats = ut + \frac{1}{2}at^2s = \frac{1}{2}(u + v)tv^2 = u^2 + 2ass = vt - \frac{1}{2}at^2$$

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# TOPIC 1: MATHEMATICAL MODELS IN MECHANICS

### 1.1 Key Modelling Assumptions

In mechanics, we simplify real-world situations using idealised models. Understanding these assumptions is crucial for solving problems correctly.

### 1.2 Standard Models

| Term | Meaning |
|---|---|
| **Particle** | An object with mass but negligible size. Can be treated as a point mass. |
| **Lamina** | A flat object with negligible thickness. |
| **Rigid Body** | Does not deform under force. |
| **Light Rod/String** | Mass can be ignored. |
| **Uniform Rod** | Mass evenly distributed; centre of mass at midpoint. |

| Term | Meaning |
|---|---|
| **Inextensible String** | Does not stretch under tension. |
| **Smooth Surface** | No friction between surfaces. |
| **Rough Surface** | Friction must be considered. |
| **Light Pulley** | Mass can be ignored; pulley is frictionless. |
| **Bead** | Particle that slides on a wire/string. |
| **Peg** | Fixed support point. |

### 1.3 Force Assumptions

- **Tension (T):** Force in a stretched string or rope, pulling towards the centre of the string.
- **Thrust/Push (P):** Compressive force pushing away from object.
- **Normal Reaction (R/N):** Perpendicular force from a surface.
- **Weight (W/mg):** Force of gravity acting downwards, $W = mg$.
- **Friction (F):** Force opposing motion or potential motion.

### 1.4 Key Principles

1. **Light String:** Tension is the same throughout.
2. **Inextensible String:** Particles attached to ends have equal acceleration.
3. **Smooth Pulley:** Tension is the same on both sides.
4. **Forces are vectors:** Can be resolved into components.

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# TOPIC 2: VECTORS IN MECHANICS

### 2.1 Vector Basics

**Scalar:** Has magnitude only (mass, time, speed)

**Vector:** Has magnitude AND direction (velocity, acceleration, force)

### 2.2 Vector Representation

**Column Form:** $\vec{a} = \binom{a_1}{a_2}$

**Unit Vectors:** $\vec{\imath} = \binom{1}{0}, \vec{\jmath} = \binom{0}{1}$

$$\vec{a} = a_1\vec{\imath} + a_2\vec{\jmath}$$

### 2.3 Magnitude and Direction

**Magnitude:** $|\vec{a}| = \sqrt{a_1^2 + a_2^2}$

**Direction:** Angle $\theta$measured from positive x-axis: $\tan\ \theta = \frac{a_2}{a_1}$

## 2.4 Vector Operations

**Addition:** $\vec{a} + \vec{b} = \begin{pmatrix} a_1 + b_1 \\ a_2 + b_2 \end{pmatrix}$

**Scalar Multiplication:** $k\vec{a} = \begin{pmatrix} ka_1 \\ ka_2 \end{pmatrix}$

**Parallel Vectors:** Two vectors are parallel if $\vec{a} = k\vec{b}$for some scalar $k$.

## 2.5 Vector Algebra

**Position Vector:** $\vec{r} = x\vec{\imath} + y\vec{\jmath}$describes position relative to origin O.

**Displacement Vector:** $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

**Example:** If $\overrightarrow{OA} = 3\vec{\imath} + 2\vec{\jmath}$and $\overrightarrow{OB} = 7\vec{\imath} - \vec{\jmath}$, find $\overrightarrow{AB}$.

**Solution:** $\overrightarrow{AB} = (7 - 3)\vec{\imath} + (-1 - 2)\vec{\jmath} = 4\vec{\imath} - 3\vec{\jmath}$

## 2.6 Relative Position and Velocity

**Relative Position:** $\vec{r}_{B \text{ relative to } A} = \overrightarrow{OB} - \overrightarrow{OA}$

**Relative Velocity:** $\vec{v}_{B \text{ relative to } A} = \vec{v}_B - \vec{v}_A$

**Example:** A particle A has velocity $\vec{v}_A = 3\vec{\imath} + 4\vec{\jmath}$and particle B has velocity $\vec{v}_B = 5\vec{\imath} + 2\vec{\jmath}$. Find the velocity of B relative to A.

**Solution:** $\vec{v}_{B/A} = \vec{v}_B - \vec{v}_A = (5 - 3)\vec{\imath} + (2 - 4)\vec{\jmath} = 2\vec{\imath} - 2\vec{\jmath}$

## 2.7 Vector Applications in Mechanics

**Velocity:** $\vec{v} = \frac{d\vec{r}}{dt}$

**Acceleration:** $\vec{a} = \frac{d\vec{v}}{dt}$

**Example:** A particle's position is given by $\vec{r} = (3t^2 + 1)\vec{\imath} + (4t - 2)\vec{\jmath}$. Find velocity and acceleration.

**Solution:** $\vec{v} = \frac{d\vec{r}}{dt} = 6t\vec{\imath} + 4\vec{\jmath}\vec{a} = \frac{d\vec{v}}{dt} = 6\vec{\imath}$

# TOPIC 3: KINEMATICS OF A PARTICLE IN A STRAIGHT LINE

## 3.1 Constant Acceleration Equations

For motion in a straight line with constant acceleration $a$:

| Equation | Formula |
|---|---|
| Velocity-Time | $v = u + at$ |
| Displacement (1) | $s = ut + \frac{1}{2}at^2$ |
| Displacement (2) | $s = \frac{1}{2}(u + v)t$ |
| Velocity-Displacement | $v^2 = u^2 + 2as$ |
| Displacement (3) | $s = vt - \frac{1}{2}at^2$ |

Where:

- $u$= initial velocity
- $v$= final velocity
- $s$= displacement
- $a$= acceleration
- $t$= time

**SI Units:**

- $s$: metres (m)
- $u, v$: metres per second ($ms^{-1}$)
- $a$: metres per second squared ($ms^{-2}$)
- $t$: seconds (s)

## 3.2 Vertical Motion Under Gravity

- Acceleration due to gravity: $g = 9.8ms^{-2}$ (downwards)
- Can treat as constant acceleration problems
- Choose a positive direction (usually up)

**Example:** A ball is thrown vertically upwards with speed 14 $ms^{-1}$. Find: a) Greatest height b) Time to return to starting point

**Solution:**

a) At greatest height, $v = 0$Taking upwards as positive ($a = -9.8$): $v^2 = u^2 + 2as0 = 14^2 + 2(-9.8)ss = \frac{196}{19.6} = 10$ m

b) For return to start, $s = 0$: $s = ut + \frac{1}{2}at^2 0 = 14t - 4.9t^2 t(14 - 4.9t) = 0 t = 0$ or $t = \frac{14}{4.9} = 2.86$ s

Total time = 2.86 s

### 3.3 Velocity-Time Graphs

**Key Properties:**

- **Gradient** = acceleration
- **Area under graph** = displacement

<TEXT>
```
        v
        |
        |    /\
        |   /  \       Area = displacement
        |  /    \
        | /      \
        |/        \
        --------------- t
```

**Example:** A car accelerates from rest at 3 ms⁻² for 10 s, travels at constant speed, then decelerates at 2 ms⁻² until it stops. Total distance = 1130 m. Find time at constant speed.

**Solution:**

Phase 1 (acceleration): $s_1 = \frac{1}{2}(0 + 30) \times 10 = 150 \text{ m} v_{max} = 0 + 3(10) = 30 \text{ ms}^{-1}$

Phase 3 (deceleration): $s_3 = \frac{1}{2}(30 + 0) \times \frac{30}{2} = 225$ m

Phase 2 (constant speed): $s_2 = 1130 - 150 - 225 = 755 \text{ m} t_2 = \frac{755}{30} = 25.2$ s

### 3.4 Problem-Solving Strategy

1. **Draw a diagram** showing the motion
2. **Choose positive direction** (stick to it!)
3. **List known quantities:** $u, v, s, a, t$
4. **Select appropriate formula**
5. **Solve and check**

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# TOPIC 4: DYNAMICS OF A PARTICLE

### 4.1 Newton's Laws of Motion

**First Law:** A particle remains at rest or continues in a straight line at constant speed unless acted upon by a resultant force.

**Second Law:** $F = ma$

Where $F$is the resultant force in Newtons (N), $m$is mass in kg, and $a$is acceleration in ms$^{-2}$.

**Third Law:** For every action, there is an equal and opposite reaction.

## 4.2 Force Diagrams

Always draw a **clear diagram** showing:

- All forces acting on the particle
- Direction of acceleration (if known)
- Choose a positive direction

**Example:** A box of mass 30 kg is pulled along a smooth horizontal floor by a force of 95 N. Find acceleration.

**Solution:**

<TEXT>

```
        R
        ↑
        |   95 N →
        □------→ Friction = 0 (smooth)
        ↓
        W = mg
```

Resolving horizontally: $F = ma95 = 30aa = 3.17\ \text{ms}^{-2}$

## 4.3 Connected Particles

When particles are connected by strings:

1. **Same magnitude of acceleration** (string inextensible)
2. **Same tension** (light, smooth pulley if applicable)

**Example:** A lift of mass 600 kg carries a man of mass 70 kg. Tension in cable is 7000 N. Find acceleration and normal reaction.

**Solution:**

Combined system (lift + man): $R_{up} - 670g = 670a7000 - 670(9.8) = 670aa = 0.648\ \text{ms}^{-2}$

Man alone: $N - 70g = 70aN = 70(9.8 + 0.648) = 731$ N

## 4.4 Particles on Inclined Planes

**Key Resolution:**

- Parallel to plane: $mg\sin\ \theta$
- Perpendicular to plane: $mg\cos\ \theta$

**Example:** A particle of mass 5 kg rests on a rough plane inclined at 30° to the horizontal. $\mu = 0.4$. Will it slide?

**Solution:**

Resolving perpendicular to plane: $R = mg\cos\ 30° = 5 \times 9.8 \times 0.866 = 42.4$ N

Maximum friction: $F_{max} = \mu R = 0.4 \times 42.4 = 16.97$ N

Force down plane (without friction): $F_{gravity} = mg\sin\ 30° = 5 \times 9.8 \times 0.5 = 24.5$ N

Since $F_{gravity} > F_{max}$, the particle will slide.

## 4.5 Friction

**Types of Friction:**

- **Static Friction (**$F \leq \mu R$**):** Prevents motion
- **Limiting Equilibrium:** $F = \mu R$(about to move)
- **Dynamic/Moving:** $F = \mu R$

**Direction:** Opposes motion or tendency to move.

**Example:** A block of mass 8 kg rests on a rough horizontal surface. A force of 30 N is applied at 20° above horizontal. $\mu = 0.3$. Find acceleration.

**Solution:**

Resolve vertically: $R + 30\sin\ 20° = 8gR + 10.26 = 78.4R = 68.14$ N

Maximum friction: $F_{max} = 0.3 \times 68.14 = 20.44$ N

Resolve horizontally: $30\cos\ 20° - F = 8a28.19 - 20.44 = 8aa = 0.97$ $\text{ms}^{-2}$

## 4.6 Pulleys

**Light, Smooth Pulley:** Tension same on both sides.

**Example:** Masses of 3 kg and 5 kg are attached to ends of a light string over a smooth pulley. Find acceleration.

**Solution:**

<TEXT>

```
        T           T
        |           |
    3kg|           |5kg
        ↓           ↓
```

For 3 kg (upward = positive): $T - 3g = 3a(1)$

For 5 kg (downward = positive): $5g - T = 5a(2)$

Adding (1) and (2): $2g = 8aa = 2.45 \text{ ms}^{-2}$

Substituting: $T = 3(9.8 + 2.45) = 36.75$ N

## 4.7 Impulse and Momentum

**Momentum:** $p = mv$(kg $\text{ms}^{-1}$)

**Impulse:** $I = Ft = mv - mu$(Ns)

**Impulse-Momentum Principle:** $I = \Delta(mv) = mv - mu$

**Example:** A ball of mass 2 kg moving at 4 $\text{ms}^{-1}$ is hit by a bat, reversing its direction at 8 $\text{ms}^{-1}$. Find impulse.

**Solution:**

Taking original direction as positive: $I = m(v - u) = 2(-8 - 4) = -24$ Ns

Impulse magnitude = 24 Ns (opposite to original direction)

## 4.8 Conservation of Linear Momentum

**Principle:** In the absence of external forces, total momentum is conserved.

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

**Example:** A railway truck A (mass 1500 kg) moving at 3 $\text{ms}^{-1}$ collides with truck B (mass 1000 kg) moving at 5 $\text{ms}^{-1}$ in the opposite direction. They couple together. Find common velocity.

**Solution:**

Taking right as positive: $1500(3) + 1000(-5) = (1500 + 1000)v4500 - 5000 = 2500vv = -0.2 \text{ ms}^{-1}$

They move at 0.2 $\text{ms}^{-1}$ to the left.

## 4.9 Impulse in Strings

When particles connected by a string separate:

- Internal impulse (string tension) acts on each particle
- Equal and opposite impulses
- Can use conservation of momentum

**Example:** Two particles P (2 kg) and Q (5 kg) connected by string, moving at 3 $ms^{-1}$ and 4 $ms^{-1}$ respectively away from each other. String becomes taut. Find common velocity.

**Solution:**

Before string taut: system has no external forces in horizontal direction.

Taking right as positive: $2(-3) + 5(4) = 7v - 6 + 20 = 7vv = 2 \text{ ms}^{-1}$

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# TOPIC 5: STATICS OF A PARTICLE

## 5.1 Equilibrium

A particle is in equilibrium if the resultant force is zero: $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots = 0$

This means:

- Resolve horizontally: $\sum F_x = 0$
- Resolve vertically: $\sum F_y = 0$

## 5.2 Resultant Forces

**Method 1: Resolution**

Break each force into horizontal and vertical components.

**Example:** Find the resultant of forces 9 N and 8 N at 60° to each other.

**Solution:**

Using cosine rule: $R^2 = 9^2 + 8^2 - 2(9)(8)\cos\ 120° R^2 = 81 + 64 + 72 = 217 R = 14.73 \text{ N}$

Using sine rule for angle: $\frac{\sin\theta}{8} = \frac{\sin\ 120°}{14.73}\theta = 27.4°$

Direction = 27.4° from 9 N force

## 5.3 Equilibrium Problems

**Strategy:**

1. Draw a clear force diagram

2. Resolve all forces horizontally and vertically
3. Set each sum equal to zero
4. Solve the equations

**Example:** A particle of weight 20 N is suspended by two strings making angles 30° and 45° with the horizontal. Find tensions.

**Solution:**

<TEXT>

```
           T₁
           \
            \ 30°
    --------●-------→ T₂
            /
           / 45°
           ↓
          20 N
```

Resolve vertically: $T_1 \sin\ 30° + T_2 \sin\ 45° = 200.5T_1 + 0.707T_2 = 20(1)$

Resolve horizontally (taking right as positive): $T_2 \cos\ 45° - T_1 \cos\ 30° = 00.707T_2 = 0.866T_1 T_2 = 1.225T_1(2)$

Substituting (2) into (1): $0.5T_1 + 0.707(1.225T_1) = 200.5T_1 + 0.866T_1 = 20T_1 = 14.6$ N

$$T_2 = 1.225 \times 14.6 = 17.9 \text{ N}$$

## 5.4 Limiting Equilibrium

When a particle is about to move: $F = \mu R$

**Example:** A particle of mass 5 kg rests on a rough horizontal plane. Coefficient of friction $\mu = 0.4$. Find the minimum force parallel to the plane needed to move it.

**Solution:**

Normal reaction: $R = mg = 5 \times 9.8 = 49$ N

Maximum friction: $F_{max} = \mu R = 0.4 \times 49 = 19.6$ N

Minimum force needed = 19.6 N

## 5.5 Particles on Inclined Planes

**Example:** A particle of mass 3 kg rests in equilibrium on a rough plane inclined at 25° to the horizontal. $\mu = 0.5$. Find the force P parallel to the plane needed to: a) Just prevent it sliding down b) Just start it moving up

**Solution:**

Resolve perpendicular to plane: $R = mg\cos\ 25° = 3 \times 9.8 \times 0.906 = 26.6$ N

Maximum friction: $F_{max} = \mu R = 0.5 \times 26.6 = 13.3$ N

a) Prevent sliding down (friction acts up): $P + F_{max} = mg\sin\ 25° P = mg\sin\ 25° - F_{max} P = 12.4 - 13.3 = -0.9$ N

Negative means no force needed (gravity alone won't cause sliding), so P = 0

b) Start moving up (friction acts down): $P = mg\sin\ 25° + F_{max} P = 12.4 + 13.3 = 25.7$ N

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# TOPIC 6: MOMENTS

## 6.1 Definition

**Moment of a Force:** Moment $= F \times d$

Where $F$ is the force and $d$ is the perpendicular distance from the pivot to the line of action.

**Unit:** Newton-metres (Nm)

**Direction:** Clockwise or anticlockwise.

## 6.2 Moment as Vector Product

If force makes angle $\theta$ with line to pivot: $M = Fd\sin\ \theta$

## 6.3 Finding Moments

**Example:** A force $\vec{F} = 7\vec{\imath} + 4\vec{\jmath}$ N acts at point (5, 3). Find moment about point (2, 1).

**Solution:**

Vector from (2,1) to (5,3): $\vec{d} = 3\vec{\imath} + 2\vec{\jmath}$

Taking moments clockwise (taking $\vec{\imath}$ as x-axis, $\vec{\jmath}$ as y-axis):

**Method 1 - Perpendicular distances:** $M = 7(2) - 4(3) = 14 - 12 = 2$ Nm clockwise

**Method 2 - Using components:** Moment from $\vec{\imath}$ component: $7 \times$(perpendicular distance in y-direction) = $7 \times 2 = 14$ clockwise Moment from $\vec{\jmath}$ component: $4 \times$(perpendicular distance in x-direction) = $4 \times 3 = 12$ anticlockwise Net moment = $14 - 12 = 2$ Nm clockwise

## 6.4 Equilibrium of Rigid Bodies

For equilibrium of a rigid body:

1. **Resultant force = 0:** $\sum F_x = 0, \sum F_y = 0$
2. **Resultant moment = 0:** $\sum M = 0$(about any point)

## 6.5 Problems Involving Beams and Rods

**Example:** A uniform beam of length 4 m and weight 20 N rests on two supports at A and B, 0.5 m from each end. A load of 15 N is placed 1 m from A. Find reactions at A and B.

**Solution:**

<TEXT>
```
A         C           D         B
|----------|-----------|---------|
0.5m      1m          1.5m      0.5m
              15 N

Beam weight = 20 N acts at centre
```

Taking moments about A: $R_B(4) - 20(2) - 15(1) = 04R_B = 40 + 15 = 55R_B = 13.75$ N

Resolving vertically: $R_A + R_B = 20 + 15 = 35R_A = 35 - 13.75 = 21.25$ N

## 6.6 Non-Uniform Rods

A non-uniform rod has centre of mass not at midpoint.

**Example:** A non-uniform rod AB of length 6 m and weight 40 N has centre of mass 2 m from A. It rests on a pivot at its midpoint. A weight of 25 N is placed 1 m from B. Find where an additional weight W must be placed from A to balance the rod.

**Solution:**

<TEXT>
```
A                    G                 B
|--------------------|----------------|
0                   2m                3m

Pivot at M (3m from A)
Taking moments about M:
Clockwise: $W \times (3 - x)$ (if weight to left of M)
Anticlockwise: $25 \times 1.5 + 40 \times 1$
For equilibrium:
$W(3 - x) = 25(1.5) + 40(1)$
$W(3 - x) = 37.5 + 40 = 77.5$
If W = 30 N:
$30(3 - x) = 77.5$
$3 - x = 2.58$
$x = 0.42$ m from M towards A
So distance from A = $3 - 0.42 = 2.58$ m
```

## 6.7 Tilting Problems

A rod begins to tilt when the normal reaction at one end becomes zero.

**Example:** A uniform rod of length 4 m and weight 30 N rests horizontally on a peg at its midpoint. A weight of 20 N is placed at one end. How far from the end can a person of weight 70 N walk before the rod begins to tilt?

**Solution:**

<TEXT>

```
Peg at midpoint M
Person walks from end P
Taking moments about M:
Clockwise: $70 \times x$ (person's moment)
Anticlockwise: $30 \times 0$ (weight through M) + $20 \times 2 = 40$
At point of tilting:
$70x = 40$
$x = 0.571$ m from M
Distance from end P = $2 + 0.571 = 2.571$ m
```

## 6.8 Ladders and Leaning Problems

**Key Principles:**

- Friction can act at ground (up to $\mu R$)
- Ladder in equilibrium if forces can be balanced

**Example:** A uniform ladder of length 5 m and weight 100 N rests against a smooth vertical wall. The foot of the ladder is 3 m from the wall. Find the reaction forces at the wall and ground.

**Solution:**

<TEXT>

```
Wall (smooth)
    |
    |
    |\
    | \
    |  \ Ladder (θ)
    |
    |
    Ground (rough)
```

$\cos\ \theta = \frac{3}{5}$, $\sin\ \theta = \frac{4}{5}$

**At wall (smooth):** Only vertical reaction $R_W$

**At ground:** Horizontal friction $F$, vertical reaction $R_G$

Resolve horizontally: $F = R_W(1)$

Resolve vertically: $R_G = 100(2)$

Take moments about ground: $R_W(4) = 100(2.5)R_W = \frac{250}{4} = 62.5$ N

Therefore $F = 62.5\text{N}$

Check limiting equilibrium at ground: $F = \mu R_G$If $\mu = 0.4$: $\mu R_G = 0.4 \times 100 = 40\text{N}$

Since $F = 62.5 > 40$, the ladder would slip with $\mu = 0.4$.

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# COMMON PROBLEM-SOLVING STRATEGIES

## 1. Drawing Clear Diagrams

**Always include:**

- All forces with arrows
- Direction of motion/acceleration
- Coordinate system
- Named points

## 2. Choosing Directions

- Choose positive direction at start
- Be consistent throughout
- For equilibrium, two directions are independent

## 3. Equations to Use

**Dynamics:**

- $F = ma$(Newton's Second Law)
- $v = u + at,\ s = ut + \frac{1}{2}at^2$, etc.
- $I = mv - mu$

**Equilibrium:**

- $\sum F_x = 0$
- $\sum F_y = 0$
- $\sum M = 0$(about any point)

## 4. Units

Always use SI units:

- Mass: kg
- Force: N
- Distance: m
- Time: s
- Acceleration: $\text{ms}^{-2}$

### 5. Checking Answers

- Does the answer make physical sense?
- Are forces reasonable?
- Is equilibrium satisfied?

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# EXAM TIPS FOR M1

## Common Mistakes to Avoid

1. **Forgetting to draw a diagram** - Always start with a diagram
2. **Wrong direction for gravity** - Downwards, always
3. **Resolving perpendicular incorrectly** - Use $mg\cos\theta$ perpendicular to slope
4. **Units** - Convert all to SI before calculating
5. **Sign conventions** - Be consistent
6. **Moments** - Use perpendicular distance, not horizontal distance

## Answer Presentation

1. **Draw a clear diagram** with all forces
2. **State assumptions** (smooth, light, etc.)
3. **Choose positive direction** and state it
4. **Show equations** before substituting numbers
5. **Give final answer** with units

## Time Management

- ~2 minutes per mark
- Start with what you can do
- Don't spend too long on one question
- Read questions carefully - they often contain clues

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# PRACTICE PROBLEMS

## Kinematics

1. A car accelerates from 20 $ms^{-1}$ to 30 $ms^{-1}$ in 5 seconds. Find: a) Acceleration b) Distance travelled in this time
2. A stone is dropped from rest into a well. The splash is heard 3 seconds later. How deep is the well? (Take $g = 10ms^{-2}$, speed of sound = 340 $ms^{-1}$)

## Forces and Motion

3. A particle of mass 6 kg is acted upon by forces $(3\vec{\imath}+2\vec{\jmath})$N and $(-\vec{\imath}+5\vec{\jmath})$N. Find the acceleration.
4. Two particles of masses 2 kg and 3 kg are connected by a light string over a smooth pulley. Find the acceleration and tension.

### Equilibrium

5. A particle is in equilibrium under forces $(3\vec{\imath}-2\vec{\jmath})$N, $(k\vec{\imath}+4\vec{\jmath})$N, and $(-7\vec{\imath}+m\vec{\jmath})$N. Find $k$and $m$.
6. A sign of weight 50 N hangs from a horizontal rod of length 2 m, supported by a string making 30° with the rod. Find tension in the string.

### Friction

7. A block of mass 10 kg rests on a rough horizontal plane. $\mu = 0.3$. Find: a) The maximum horizontal force before it moves b) If a force of 40 N is applied, find acceleration

### Momentum

8. A ball of mass 0.5 kg moving at 8 ms$^{-1}$ hits a wall and rebounds at 6 ms$^{-1}$. Find: a) Change in momentum b) Impulse on the ball

### Moments

9. A non-uniform beam AB of length 6 m weighs 80 N. Its centre of mass is 2 m from A. It rests on supports at A and at a point 1 m from B. Find the reactions at the supports when a load of 100 N is placed 1 m from A.

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# ANSWERS TO PRACTICE PROBLEMS

### 1. Kinematics

a) $a = \frac{30-20}{5} = 2\text{ms}^{-2}$

b) $s = \frac{(20+30)}{2} \times 5 = 125\text{m}$

### 2. Stone Drop

Time to fall: $t_1 s = \frac{1}{2} g t_1^2 \Rightarrow t_1 = \sqrt{\frac{2s}{10}}$

Time for sound: $t_2 = \frac{s}{340}$

Total: $t_1 + t_2 = 3\sqrt{\frac{s}{5}} + \frac{s}{340} = 3$

Solving: $s \approx 41\text{m}$

## 3. Acceleration

Resultant force = $(3-1)\vec{i} + (2+5)\vec{j} = 2\vec{i} + 7\vec{j}\text{N}$ $a = \frac{F}{m} = \frac{2\vec{i}+7\vec{j}}{6} = \frac{1}{3}\vec{i} + \frac{7}{6}\vec{j}$ ms$^{-2}$

## 4. Pulley System

Taking 3 kg moving down as positive: $3g - T = 3a(1)T - 2g = 2a(2)$

Adding: $g = 5aa = \frac{9.8}{5} = 1.96$ ms$^{-2}$

From (2): $T = 2(9.8 + 1.96) = 23.5\text{N}$

## 5. Equilibrium

For equilibrium: Sum of forces = 0 $3 + k - 7 = 0 \Rightarrow k = 4 - 2 + 4 + m = 0 \Rightarrow m = -2$

## 6. Sign and String

<TEXT>

```
         T
        /
       / 30°
─────●───── 2m ─────
     |
     50 N
```

Resolving vertically: $T\sin\ 30° = 50T = \frac{50}{0.5} = 100$ N

## 7. Friction

a) $R = mg = 98\text{N}$ $F_{max} = \mu R = 0.3 \times 98 = 29.4\text{N}$ Maximum horizontal force = 29.4 N

b) Since 40 N > 29.4 N, block accelerates: $40 - 29.4 = 10aa = 1.06$ ms$^{-2}$

## 8. Momentum

Taking direction away from wall as positive: $\Delta p = m(v - u) = 0.5(-6 - 8) = -7$ kg ms$^{-1}$

Impulse = -7 Ns (opposite to original direction)

## 9. Moments

<TEXT>

```
A                      G           C        B
|----------------------|----------|--------|
0                     2m          4m       6m
          100 N                 Support
          ↓
      Beam weight 80 N
```

Taking moments about A: $R_C \times 5 = 100 \times 1 + 80 \times 25R_C = 100 + 160 = 260R_C = 52$ N

Resolving vertically: $R_A + R_C = 100 + 80 = 180R_A = 180 - 52 = 128$ N